EW physics at Z-pole

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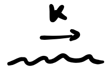
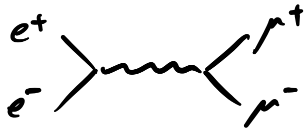
We will study processes in a collider involving e^+e^- collisions. For $E \ll m_Z$, we can employ the Fermi theory to study the process. However, at $E \sim m_Z$ we shall use the full SM.

We will be interested in $e^+e^- \to \mu^+\mu^-$ at $s \simeq m_Z^2$. We immediatelly encounter the issue that the Z propagator



$$= \frac{i}{k^2 - m_Z^2}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2}\right)$$

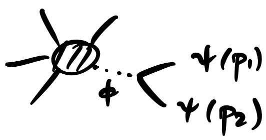
diverges for $k^2 = m_Z^2$. Then, the amplitude



seems to diverge at $(p_{e^-}+p_{e^+})^2 = m_Z^2$.

- The solution is that the "true" Z boson propagator is not the one above.

A generic process in a theory with a scalar ϕ of mass m & fermions ψ with mass μ and Yukawa $\bar\psi\psi\phi$,



$\hookrightarrow$ divergence at $(p_1+p_2)^2 = m^2$.

Notice that this is only possible if $m > 2\mu$,

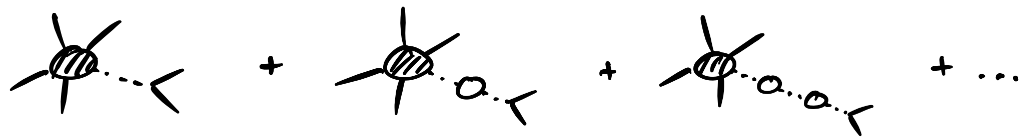
$$(p_1+p_2)^2 = 2\mu^2 + 2(E_1E_2 - \vec{p}_1\cdot\vec{p}_2)$$

and

$$\begin{aligned} E_1^2E_2^2 &= (|\vec{p}_1|^2+\mu^2)(|\vec{p}_2|^2+\mu^2) \\ &= \mu^4 + \mu^2(|\vec{p}_1|^2+|\vec{p}_2|^2) + |\vec{p}_1|^2|\vec{p}_2|^2 \\ &= (\mu^2+|\vec{p}_1||\vec{p}_2|)^2 + \mu^2(|\vec{p}_1|-|\vec{p}_2|)^2 \\ &\geq (\mu^2+|\vec{p}_1||\vec{p}_2|)^2 \end{aligned}$$

$$\Rightarrow (p_1+p_2)^2 \geq 4\mu^2 + 2(|\vec{p}_1||\vec{p}_2| - |\vec{p}_1||\vec{p}_2|\cos\theta) \geq 4\mu^2$$

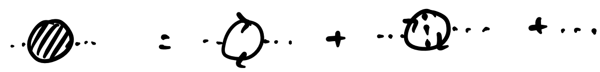
So the divergence appears if and only if the particle can decay, i.e. is unstable.
The solution is also related to this.
We need to remember to add the radiative corrections



The all order two-point function is

$$\frac{i}{A} \qquad \frac{i}{A}B\frac{i}{A} \qquad \frac{i}{A}B\frac{i}{A}B\frac{i}{A}$$

where $A = k^2 - m^2$

and $-iB =$ 

Summing the full series,

$$\bullet\bullet\bullet\bullet = \frac{i}{A}\sum_{n=0}^{\infty}\left(B\cdot\frac{1}{A}\right)^n = \frac{i}{A-B}$$

which does not diverge as $A \to 0$!
This is a trick, we cannot resum it if we

are outside the convergence radius.

The correct expression is defined in the Euclidean, resumed, and continued to Mink.

The true 2-pt fn is

$$\frac{i}{A-B} = \frac{i}{k^2-m^2-B(k^2)}$$

So, no matter how small the loop correction B is, at $k^2=m^2$ the "tree" level term vanishes and B dominates.

- Notice that this is a solution of our problems only if B has an imaginary part.

- We can use the optical theorem.

The S-matrix is unitary,

$$S^\dagger S = 1.$$

This means that the transition matrix T, $S = 1 + iT$, obeys

$$-i(T-T^\dagger) = T^\dagger T$$

Consider a generic "a→b" process,

$$i\langle b|T|a\rangle = i\mathcal{A}(a\to b)\,(2\pi)^4\delta^4(p_a-p_b)$$

using

$$\mathbb{1} = \sum\!\!\!\!\!\!\int \prod_i \frac{d^3q_i}{(2\pi)^3 2E_i}\,|q_i\rangle\langle q_i|$$

we get

$$\langle b|T^\dagger \mathbb{1}\, T|a\rangle = \sum\!\!\!\!\!\!\int \underbrace{\langle b|T^\dagger|q_i\rangle}_{(\langle q_i|T|b\rangle)^*}\langle q_i|T|a\rangle$$

$$= \sum\!\!\!\!\!\!\int \mathcal{A}(a\to q_i)\,\mathcal{A}^*(b\to q_i)\,(2\pi)^{4+4}\delta^4(p_a-q_i)\,\delta^4(p_b-q_i)$$

$$= (2\pi)^4\delta^4(p_a-p_b)\underbrace{\sum\!\!\!\!\!\!\int \widetilde{dq_i}\,(2\pi)^4\delta^4(p_a-p_{q_i})}_{\int d\Phi_n}\mathcal{A}(a\to q_i)\,\mathcal{A}^*(b\to q_i)$$

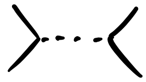
So the optical theorem gives

$$-i\Big(\mathcal{A}(a\to b)-\mathcal{A}^*(b\to a)\Big) = \sum_I \int d\Phi_I\, \mathcal{A}(a\to I)\,\mathcal{A}^*(b\to I)$$

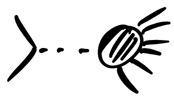
where I denotes any intermediate state.

- Now consider two particles $\alpha,\bar\alpha$ and $\beta,\bar\beta$, with coupling ε to ϕ.

We apply the optical theorem to $\alpha\bar{\alpha} \to \beta\bar{\beta}$ scattering. The amplitude is just

$$i\mathcal{A}(\alpha\bar{\alpha} \to \beta\bar{\beta}) =$$  $$= i(i\varepsilon)^2 \frac{1}{k^2 - m^2 - \mathrm{Re}B - i\,\mathrm{Im}B} + \mathcal{O}(\varepsilon^4)$$

and the inclusive amplitude is

$$i\mathcal{A}(\alpha\bar{\alpha} \to I) = i\mathcal{A}(\beta\bar{\beta} \to I) =$$ 

$$= i(i\varepsilon)\frac{1}{k^2 - m^2 - B} \cdot \mathcal{A}(\phi \to I)$$

The optical theorem leads to

$$-i(i\varepsilon)^2 \left[\frac{1}{k^2 - m^2 - B} - \frac{1}{k^2 - m^2 - B^*} \right] =$$

$$= (i\varepsilon)(-i\varepsilon) \sum\!\!\!\!\!\!\int d\Phi_n \, |\mathcal{A}(\phi \to I)|^2 \left| \frac{1}{k^2 - m^2 - B} \right|^2$$

$$\Rightarrow \quad i \frac{2i\,\mathrm{Im}B}{|k^2 - m^2 - B|^2} = \frac{1}{|k^2 - m^2 - B|^2} \underbrace{\sum\!\!\!\!\!\!\int_I d\Phi_n \, |\mathcal{A}(\phi \to I)|^2}_{= 2m\Gamma_{tot}}$$

$$\Rightarrow \quad \mathrm{Im}B = -m\Gamma_{tot}$$

So the imaginary part of B is nonzero, strictly negative, and indeed related to the decay width.

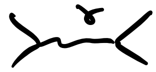
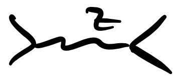
Then, the singularity is moved away from the real axis and never reached for physical energies.

Notice that T contains information of all particles coupling to the resonance. Therefore, it is possible to infer couplings even if some states are not measured.

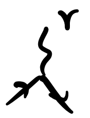
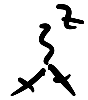
- $e^+e^- \to \mu^+\mu^-$

Back to our goal. $e^+e^- \to e^+e^-$ is significantly more involved due to t-channel exchanges. $e^+e^- \to$ hadrons will be the subject when discussing QCD.

The amplitude is given by

$$i\mathcal{A} = i\mathcal{A}_\gamma + i\mathcal{A}_Z =$$  $$+$$ 

with

 $$= -ie\gamma^\mu \,,$$  $$= i\frac{g}{4c_W}(g_V - g_A\gamma^5)\gamma^\mu$$

 $$= \frac{i}{p^2 - m_Z^2 + i m_Z \Gamma_Z}\left(-\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m_Z^2}\right)$$

 $$= -\frac{i}{p^2 + i\varepsilon}\,\eta_{\mu\nu}$$

We compute the cross section as a function of the scattering angle θ, $\frac{d\sigma}{d\cos\theta}$.

Neglecting lepton masses, the result is

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha_{em}^2}{2s}\left[A(1+\cos^2\theta) + B\cos\theta\right]$$

with

$$A = \overset{\gamma^2}{1} - \overset{\gamma-Z}{\frac{8\sqrt{2}}{e^2} g_V^2\, s\, \mathrm{Re}\left[G_F K(s)\right]} + \overset{Z^2}{\frac{32}{e^4}(g_V^2 + g_A^2)\, s^2 \left|G_F K(s)\right|^2}$$

$$B = \underset{\gamma-Z}{-\frac{16\sqrt{2}}{e^2} g_A^2\, s\, \mathrm{Re}\left[G_F K(s)\right]} + \underset{Z^2}{\frac{16\cdot 16}{e^4} g_V^2 g_A^2\, s^2 \left|G_F K(s)\right|^2}$$

with

$$\frac{G_F\, m_Z^2}{m_Z^2 - s - i m_Z \Gamma_Z} \equiv G_F\, K(s) = \frac{g^2}{4\sqrt{2}\, c_W^2} \frac{1}{m_Z^2 - s - i m_Z \Gamma_Z}$$

- In QED, $A = 1$, $B = 0$. This is because QED preserves parity, and the P-odd term $\cos\theta$ vanishes.

Under P, μ^- momentum changes sign, so $\theta \to \theta + \pi$ & $\cos\theta \to -\cos\theta$.

Weak interactions break parity, and this affects the differential distribution

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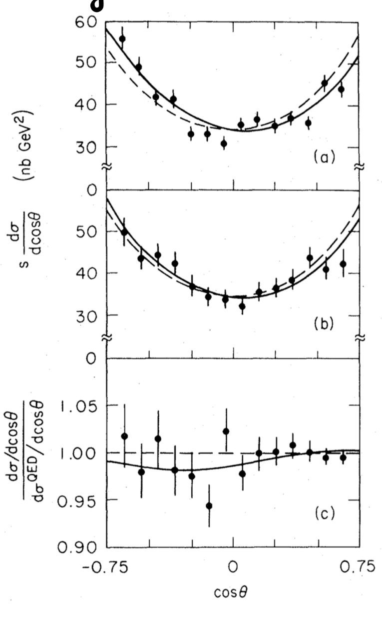

$e^+e^- \to \mu^+\mu^-$ @ $\sqrt{s} = 29\ \text{GeV}$

$e^+e^- \to \tau^+\tau^-$

$$\frac{e^+e^- \to \mu^+\mu^- |_{SM}}{e^+e^- \to \mu^+\mu^- |_{QED}}$$

- Instead of using "A" and "B", we can define two observables: the total cross section and the forward-backward asymmetry. Namely,

$$\sigma_F = \int_0^1 dc_\theta \frac{d\sigma}{dc_\theta} = \frac{\pi\alpha^2}{2s}\left(\frac{4}{3}A + \frac{1}{2}B\right)$$

$$\sigma_B = \int_{-1}^0 dc_\theta \frac{d\sigma}{dc_\theta} = \frac{\pi\alpha^2}{2s}\left(\frac{4}{3}A - \frac{1}{2}B\right)$$

$$\Rightarrow \qquad \sigma_{tot} = \frac{4\pi\alpha^2}{3s} A = \sigma_F + \sigma_B$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{8}\frac{B}{A}$$

- At low energy, $G_F K(s) \simeq 1$. The result coincides with the one in the Fermi theory. Since $G_F s \ll 1$, the effect of weak interactions is small compared with QED, and the leading contribution comes from the interference

$$A = 1 - \frac{8\sqrt{2}}{e^2} g_V^2 G_F s + \dots$$

$$B = - \frac{16\sqrt{2}}{e^2} g_A^2 G_F s + \dots$$

From $e^+e^- \to \mu^+\mu^-$ was possible to complete the phenomenological study of g_V^ℓ, g_A^ℓ, and resolve the 2-fold ambiguity. This lead to the first measurement of s_W^2.

. With $s_W^2 \simeq 0.23$, together with $v \simeq 246\,\text{GeV}$ from μ^- decay and $e = 0.3$ from atomic physics, since

$$g s_W = g' c_W = e \longrightarrow g = 0.63, \; g' = 0.34$$

from which we could <u>predict</u> the mass of the W and Z bosons

$$m_W = \frac{1}{2} g v \simeq 80\,\text{GeV}$$

$$m_Z = \frac{m_W}{c_W} \simeq 90\,\text{GeV}$$

- A "Large Electron Positron" collider (LEP) was build to study e^+e^- collisions near $\sqrt{s} \sim 90\,\text{GeV}$ in order to study the Z-boson properties.

Close to the Z mass, the process is dominated by the Z-exchange, since the propagator

$$\frac{1}{s - m_Z^2 + i m_Z \Gamma_Z} \sim \frac{1}{i m_Z \Gamma_Z}$$

is enhanced by the smallness of Γ_Z, as it is of loop size.

Close to the resonance,

$$s = m_Z^2 (1 + \varepsilon) \quad , \quad \varepsilon \ll 1$$

we get

$$A \simeq \frac{g^4}{e^4 c_W^4} (g_V^2 + g_A^2) \frac{1}{\varepsilon^2 + \Gamma_Z^2 / m_Z^2}$$

$$B \simeq \frac{8 g^4}{e^4 c_W^4} g_V^2 g_A^2 \frac{1}{\varepsilon^2 + \Gamma_Z^2 / m_Z^2}$$

$$\Rightarrow \quad \sigma_{tot} = \frac{g^4}{12 \pi c_W^4} (g_V^2 + g_A^2)^2 \frac{1}{\Gamma_Z^2 + m_Z^2 \varepsilon^2}$$

where $\varepsilon = \dfrac{s - m_Z^2}{m_Z^2}$.

By measuring σ_{tot} as a function of s we can extract

1) peak position : m_Z

2) half-width : Γ_Z

3) σ_{peak} : s_W

Experimentally, it leads to very accurate measurements

$$m_Z = 91.1876(21) \text{ GeV}$$
$$\Gamma_Z = 2.4952(23) \text{ GeV}$$
$$s_W^2 = 0.23116(12)$$